Endogenous Risks and Learning in Climate Change Decision Analysis

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Abstract. We analyze the effects of risks and learning on climate change decisions. Using a new two-stage, dynamic, climate change stabilization model with random time horizons, we show that the explicit incorporation of ex-post learning and safety constraints induces risk aversion in ex-ante decisions. This risk aversion takes the form in linear models of VaR- and CVaR-type risk measures. We also analyze extensions of the model that account for the possibility of nonlinear costs, limited emissions abatement capacity, and partial learning. We find that in all cases, even in linear models, any conclusion about the effect of learning can be reversed. Namely, learning may lead to either less- or more restrictive ex-ante emission reductions depending on model assumptions regarding costs, the distributions describing uncertainties, and assumptions about what might be learned.

We analyze stylized elements of the model in order to identify the key factors driving outcomes and conclude that, unlike in most previous models, the quantiles of probability distributions play a critical role in solutions.

Keywords: stochastic nonsmooth optimization, climate stabilization, learning, catastrophic risk.

1 Introduction

Early climate change mitigation analysis tended to frame the climate change problem as a hit-or-miss type of decision making situation, in which a single policy choice is made about appropriate emissions reductions over time. Increasingly, analysis explicitly recognizes that the problem is more accurately described as sequential decision making under uncertainty, with the anticipation that new information will be acquired over time. For example, one reflection of this orientation can be found in discussions of climate change policies [14] that are framed as a choice between acting now or waiting until we know more about the problem [17], [27], [28]. This is a natural framing of the problem given its key characteristics involving uncertainty, irreversibility, and the potential for learning. Emissions of greenhouse gases (GHG) associated with the production and consumption of goods and services lead to long-lived atmospheric concentrations (stocks) of these pollutants that alter Earth’s climate. On one hand, postponing the reduction of GHG emissions may lead to costly and potentially irreversible climate-related impacts such as reorganization of large-scale ocean circulation patterns or increased frequency of extreme weather-related events. On the other hand, undertaking
emission abatements now risks potentially irreversible investments that may turn out to be unnecessary if climate change is less severe than expected.

In economics literature, the importance of learning was first discussed in connections with irreversible investments in 1974 in Arrow and Fisher [2] and Henry [13] without an overall two-stage model being formulated. Arrow and Fisher [2], Henry [13], and Chichilnisky and Heal [3] have concluded that when future damages are uncertain and irreversible, the ability to learn should lead to more active ex-ante emission reductions. On the other hand, irreversibility of capital may lock the economy in a wasteful use of resources. Viscusi and Zeckhauser [25], Dixit and Pindyck [5], Ulph and Ulph [24], and Pindyck [21] showed that the ability to learn in this case should lead to less active ex-ante emission reduction. These competing effects imply that the net effect of learning on ex-ante decisions is an empirical question. Nordhaus [20] and Kolstad [16] examined the effects of learning by using empirically-calibrated integrated assessment models. They concluded that, in fact, learning has insignificant effects on ex-ante abatement policies because the damage losses are not severe enough. A reason for this is that in the most integrated climate and economics models, climate changes are considered as if they occur continuously and as if they can eventually be reversed through ex-post adjustments [28]. These models also use average damages (i.e., they cannot properly capture the effects of abrupt climate change [1] and catastrophic risks [8], [19], [28]). A paper by Fisher and Narain [11] analyzed a two-period model with risk characterized by a parameter introducing high or low climate change damages. Because overall impacts are evaluated by using expected values, the effects of capital irreversibility dominate catastrophic damages in a similar way to other models. Epstein [7] demonstrated that the effects of learning on ex-ante decision depend in general on convexity or concavity of marginal costs, which are very restrictive for climate change policy analysis [24].

In this paper we take a different approach. Instead of using expected damages we explicitly introduce safety constraints by formulating the climate change problem within the two-stage framework of stabilization. We develop a two-stage dynamic STO model with random time horizons and deliberately analyze only stylized linear versions of this model. Our simple analytical analysis allows to avoid otherwise endless number of computational experiments in order to identify effects of various driving forces. In particular, we show that the combination of safety constraints and perspectives of learning in linear models induces potentially strong risk aversion among ex-ante decisions that is characterized by quantile based VaR (Value at Risk) and CVaR (Conditional Value at Risk) risk measures common in the risk literature. As a result we show that, even with a linear net cost function, learning may lead to either less or more restrictive emission reductions, depending on mitigation costs and probability distributions describing key uncertainties.

The paper is organized as follows. Section 2 develops the general model, characterizing climate change risk by the probability of total atmospheric
CO₂ concentrations exceeding a vital random threshold associated with potential ranges of global temperature. It outlines a new type of two-stage dynamic climate-change stabilization STO model with random durations of stages. In general, this model can be solved only numerically and therefore the key factors driving results are difficult to identify. For these reasons, we analyze only stylized aspects of the model; these provide a clearer picture of the various driving forces and show why the ability to learn in the future can lead to either less restrictive or more restrictive ex-ante abatement policies today.

Section 3 presents our basic simplified model. It uses a very simple linear two-stage STO model to illustrate that the results from empirical models can be contradictory because optimal solutions depend on complex nonsmooth interactions among ex-ante and ex-post decisions, costs, and probability distributions. In particular, solutions contain potentially strong risk aversion characterized by quantile-based risk measures that are used for regulating the safety of nuclear plants and insolvency of insurance companies, but also in financial applications, extremal value theory [6], and catastrophic risk management [8].

Section 4 analyzes three extensions of the basic model designed to investigate the consequences of non-linear costs, limitations on stage two reductions (or adaptive capacity), or incomplete learning. Results emphasize again the importance to optimal solutions of quantiles of probability distributions characterizing key uncertainties, and also more strongly and even unconditionally require ex-ante anticipative emissions reduction in addition to ex-post reductions.

A more realistic but still linear dynamic two-stage climate change stabilization STO model is analyzed in Section 6. Similar to Section 3, the explicit incorporation of ex-ante and ex-post decisions induces risk aversions characterized by a dynamic version of a CVaR type risk measure. This may create the misleading impression that a truly risk-based policy analysis has been carried out and, without the explicit introduction of adaptive capacity and additional safety constraints, may provoke a catastrophe. In conclusion, Section 6 emphasizes the importance of the proper models, explicit treatment of uncertainty and risks, more realistic accounting for uncertainty, and robust decisions.

2 Endogenous Climate Change Risk: A General Model

Climate-change integrated assessment models (see e.g., [20]) incorporate economic and geophysical processes that link economic growth with the accumulation of GHG emissions in the atmosphere. The accumulation of CO₂ emissions is the main driving force behind global climate change over multi-decade timescale. The process involves complex interactions between the atmosphere, the terrestrial biosphere, and the oceans. Current integrated as-
Assessment models use different carbon-cycle models for computing changes in atmospheric concentrations $M(t)$ resulting from CO$_2$ emissions $e(t)$ [23]. In general, these models are of the form

$$M(t + 1) = f(M(t), e(t), \beta), t = 0, 1, 2, ...,$$

where $\beta$ is a vector of model parameters. Values $M(t)$ are used in integrated assessment models to compute the increase in the global average temperature as a smooth function of $M(t)$, and damages are typically computed in the form of smooth functions of this temperature increase.

This approach to modeling damages has several weaknesses. For example, a serious underestimation of damages may result from the use of global average temperature as a measure of climate change. Changes in the frequency of extreme weather-related events (e.g., floods, droughts, storms, heat waves) may be more important and may be non-linearly related to changes in the mean temperature. In addition, it has been proposed that beyond particular climate change thresholds, singular catastrophic events may be triggered that will have widespread consequences such as changes in ocean circulation, or disintegration of ice sheets [19].

We therefore develop an alternative approach to modeling climate change damages that treats threshold-type risks explicitly and that also accounts for uncertainty and its potential resolution over time. We first present a brief qualitative summary of the model, and then its mathematical form.

We assume that emissions in the absence of climate policy (specifically, emissions mitigation policy) depend on a wide range of uncertainties in future socio-economic development paths, technological progress, and lifestyles which together can be grouped as scenarios. Furthermore, emissions are also affected by explicit mitigation policies that might be adopted with uncertain costs. Emissions lead to uncertain accumulation of atmospheric concentrations and uncertain climate changes (and therefore damages). We introduce risk by assuming that there is a safety constraint in the form of an atmospheric stabilization target; i.e., a level of greenhouse gas concentration we wish to avoid exceeding with some level of confidence, based on an assumed level of aversion to the risk of incurring serious damages. Introducing stabilization in the form of a chance constraint is consistent with the uncertainty assumed in the problem: as long as uncertainty in the climate system persists, any emissions path will yield at best a range of possible outcomes. Thus, policies can only limit the chance of exceeding any particular target, and the acceptable risk must be defined a priori.

We introduce learning by assuming there are two stages. In the first, decision makers face full uncertainty. After a particular (and uncertain) length of time, some new information about uncertain parameters is revealed, and the second stage begins, at which point emission mitigation policies may be adjusted. The problem is to choose the emissions reduction policies for both stages such that mitigation costs are minimized and the safety constraint is achieved with the desired level of confidence.
This model of concentration-stabilization with learning can be defined mathematically as follows. Let emissions \( e(t) \) in (1) depend on scenario uncertainties \( \omega \) and policy variables \( x \). In this paper, we assume that scenarios are characterized as random variables defined in a probability space \( \Omega, \omega \in \Omega \), with a probability measure \( P(d\omega) \). Thus, for \( \Omega = \{1, ..., N\} \),

\[
P(d\omega) = p(s) := P[\omega = s], \sum_{s=1}^{N} p(s) = 1.
\]

Frequently we do not indicate the dependence of random variables on \( \omega \) if this is clear from the context.

Let us denote by \( L(\omega) \) the uncertain target level of CO\(_2\) concentration in the atmosphere. If \( M(t, x, \omega) \) denotes the mitigated CO\(_2\) concentration, then the main problem can be formulated as the choice of cost-efficient emission-reduction path that satisfies probabilistic safety constraints on vital but uncertain levels of concentrations

\[
P[M(t, x, \omega) \leq L(\omega), t = 1, T] \geq 1 - \gamma,
\]

where \( \gamma \) is a risk factor, \( 0 \leq \gamma < 1 \), \( T \) is a time horizon that may also be uncertain. The violation of these constraints can be regarded as a catastrophic collapse. In the insurance industry, constraints of type (2) regulate risk reserves to prevent insolvency. The typical approach to choosing \( \gamma \) in this industry is not based primarily on evaluating potential damages, but rather on limiting the chance that the insolvency may occur, say, to only once in 800 years, \( \gamma = 1/800 \). Similarly, the major failure of a nuclear plant is allowed once in \( 10^7 \) years, \( \gamma = 10^{-7} \). Note that these are expected time horizons and therefore there is the possibility that events may occur at any time.

The abrupt climate change in (2) is modeled by random \( L(\omega) \), which is revealed as a shock at random moment \( \tau(x, \omega) \), which may also depend on \( x \). Despite a smooth and even linear dependence of function \( M(t, x, \omega) \) on \( x \), the left-hand side of (2) is, in general, a nonsmooth and often even a discontinuous risk function \([9],[10],[18]\). Endogenous catastrophic collapse or the vulnerability of analyzed socio-economic and environmental system is modeled as a violation of constraint (2). In general, the learning may not reveal full information but perhaps only shift ranges of probability distributions. The learning may also not occur at \( \tau \leq T \), or it may occur very close to \( T \). Because the inertia of the system may not allow constraints (2) to be fulfilled quickly, the probability of a catastrophe conditional on revealed information may drop rapidly below the vital level \( \gamma \) (i.e., constraint (2) emphasizes the importance of proper ex-ante actions).

Learning is reflected by specifying a two-stage dynamic STO framework with random time horizon. At stage 1, the emission-reduction path is defined by ex-ante decisions \( x(t), t = 1, 2, ..., \) until a random time moment \( \tau \) when new information is revealed. The new information may also include a new critical time horizon \( T(\omega) \) for stage 2 ex-post emission reductions \( x(t, \omega), t = \tau + 1, ..., T(\omega) \). A new \( T(\omega) \) can be shorter or longer than initial \( T \), what may essentially affect adaptive capacity of the system. In general, the ex-ante policy \( x(t), t \leq \tau \), includes components for choosing appropriate feasible level
of this capacity. The problem is to minimize total emission reduction cost subject to constraints (2).

The resulting model can only be solved numerically. Here, instead of using numerical simulations, we take a different approach. In the following sections we formulate various stylized elements of the model and evaluate them analytically. This allows us to keep the discussion on a simple level, which provides a clear picture of the driving forces, their appropriate treatments and potential outcomes.

3 A Basic Model with Linear Cost Functions

The first stylized version of the general model we analyze aims to simplify it to the maximum extent in order to establish which types of solutions are possible even in the most basic case. Accordingly, we assume there are only two periods; that mitigation costs are certain and linear in reductions; that all other uncertainties can be collapsed into a single variable; and that the learning that takes place before stage two begins completely resolves this uncertainty. The single variable treated as uncertain is total emissions reductions over both periods, and the constraint is also expressed in terms of this variable. A constraint on minimum emissions reduction can be thought of as a concise way to represent several factors (and their uncertainties) that come into play in meeting a target based on environmental outcomes such as atmospheric concentrations, global average temperature levels, or particular impacts. For example, emissions reductions required to meet a target will depend on the target itself (i.e., whether a concentration or temperature target is high or low), on unmitigated reference emissions (because the absolute size of required emissions reductions will depend on the magnitude of uncontrolled emissions in the reference case), and on the system mapping emissions to environmental outcomes (e.g., parameters of the carbon cycle or climate system). Uncertainty in total required emissions reductions can be thought of as reflecting uncertainty in one or more of these different factors. Because all uncertainty is resolved before stage two begins, a chance constraint (as in the general problem) is unnecessary; the constraint can always be met with full certainty.

In Section 3.1 we specify the model mathematically and present its solution, and in Section 3.2 discuss the effects of learning (in comparison to an identical problem with uncertainty but no learning, or with perfect information). In Section 3.3 we provide a discussion of the results.

3.1 Two-stage model

A stylized concentration-stabilization problem with learning can be formulated as follows: assume that there are only two time intervals or periods \( t = 1, 2 \). Define by \( x_t, x_t \geq 0, t = 1, 2 \), a feasible level of emission reduction
that can be chosen in period $t$; $C_1, C_1 > 0$, is the known expected abatement cost per unit of emission reduction in period $t$; $\theta(\omega)$ is the uncertain target value of cumulative emission reductions for two periods. In this problem formulation, $\theta(\omega)$ serves as the safety constraint, $x_1 + x_2 \geq \theta(\omega)$. As discussed above, uncertainty in $\theta(\omega)$ can be thought of as reflecting uncertainty in any one of several different factors.

Assume uncertainty in $\theta(\omega)$ is resolved between periods 1 and 2. The ex-ante decision $x_1$ is made before the uncertainty in $\theta$ is resolved, whereas the ex-post decision $x_2$ is based on known $\theta$, i.e., $x_2$ is a function of $\theta$, $x_2(\theta)$. Assume that the ability to fulfill risk constraint $x_1 + x_2 \geq \theta(\omega)$ in period 2 is unbounded (the impacts of this rather unrealistic assumption are analyzed in Section 4. The problem is formulated as the minimization of total expected linear costs

$$C_1 x_1 + C_2 E x_2(\theta)$$

subject to safety constraints

$$x_1 + x_2(\theta) \geq \theta$$

for all $\theta$. This is a classical two-stage STO problem [4], [10], [15], [26]. Clearly, if the ex-ante decision $x_1$ is irreversible, then the optimal period 2 decision is $x_2(\theta) = \max \{0, \theta - x_1\}$, that is, it depends non-smoothly on period 1 decision $x_1$ (path-dependence) and $\theta$, providing potentially strong cross-period random interactions among decisions. Optimal period 1 decision $x_1^*$ solves the stochastic minimax problem: minimize

$$F(x) = C_1 x + C_2 E \max \{0, \theta - x\}, x \geq 0. \quad (5)$$

**Remark 2.** Although the initial model (3)-(4) is linear in $(x_1, x_2)$, the introduction of ex-post decision $x_2(\omega)$ induces risk aversion among ex-ante decisions that is defined by implicit non-smooth (in general) function (5). The following Proposition summarizes the solution and some important facts about stochastic minimax problem (5). It shows that the induced risk attitudes are characterized by VaR (critical quantile) and CVaR risk measures [22] allowing further to derive main conclusions regarding effects of learning. In extremal value theory [6], CVaR is also known as Mean Shortfall and Mean Excess Loss.

**Proposition.** Assume that $H(z) = P[\theta \leq z]$ is a continuously differentiable function.

(i) $F(x)$ is a strictly convex continuously differentiable function.

(ii) If $C_1 > C_2$, then $x_1^* = 0$ and $x_2^*(\theta) = \theta$. If $C_1 < C_2$, then the necessary and sufficient condition for optimal $x^*$ reads: $x^*$ is the quantile satisfying equation

$$P[\theta \geq x] = C_1 / C_2.$$  

(iii) The optimal value $F(x^*)$ has two important representations involving the expected cost under perfect information, the expected value of perfect
information and the CVaR risk measure:
\[
F(x^*) = C_1 \bar{\theta} + C_1 E[x^* - \theta | \theta \leq x^*] = C_2 E\theta I(\theta \geq x^*),
\]
where \( E[\cdot | \cdot] \) denotes the conditional expectation, the indicator function \( I(\theta \geq x) = 1 \) if \( \theta \geq x \) and \( I(\theta \geq x) = 0 \) otherwise.

Let us outline the proof.
(i) The convexity of \( F(x) \) follows from the convexity of function \( \max \{ 0, \theta - x \} \) which is preserved under the expectation operation. The strict convexity and continuous differentiability of \( F(x) \) follow from the continuous differentiability of \( H(z) \).
(ii) The minimization of \( F(x) \) is a specific case of so-called stochastic minimax problems [9]. From the general results (see, e.g., page 416 of [10]) it follows that \( F'(x) = C_1 - C_2 P[\theta \geq x] \). From \( C_1 < C_2 \), it follows that \( F'(0) < 0 \), i.e., \( x^* > 0 \) (assuming \( x^* = 0 \) we can derive contradiction with assumption \( C_1 < C_2 \) for small \( x \)). As \( F(x) \) is a strictly convex function, it follows that (6) is indeed a necessary and sufficient optimality condition.
(iii) The first representation in (7) follows from (6) and the following rearrangements:
\[
F(x^*) = C_1 x^*_1 + C_2 E \max \{ 0, \theta - x^* \} = C_1 x^* + C_2 E[\theta - x^* | \theta \geq x^*] P[\theta \geq x^*] = C_1 x^* + C_2 (E[\theta - x^* | \theta \leq x^*] - E[\theta - x^* | \theta \leq x^*]) = C_1 x^* + C_2 E[x^* - \theta | \theta \leq x^*]
\]
The second representation in (7) follows from (6) and \( E \max \{ 0, \theta - x^* \} = E(\theta \geq x^*) - x^* P(\theta \geq x^*) \).

Remark 3. The critical quantile in (6) defines the VaR risk measure; i.e., it indicates the magnitude of emission reduction in stage 1 that, with probability \( 1 - C_1 / C_2 \), will be sufficient to meet the safety constraint with no additional reduction required in stage 2. The first term \( C_1 \bar{\theta} \) in (7) represents the expected cost under perfect information. The second term represents the expected value of perfect information; i.e., the value of learning the true value of \( \theta \) before stage 1, rather than after stage 1 and before stage 2.

The second equation in (7) defines the CVaR risk measure; i.e., the expected value of abatement costs that will be necessary in stage 2 if emissions reductions in stage 1 are not sufficient to meet the safety constraint. For some distributions it is possible to derive \( x^* \) from (6) explicitly. If \( \theta \) is uniformly distributed on \([a, b]\), then it is easy to see that \( x^* = \frac{C_1}{C_2} a + (1 - \frac{C_1}{C_2}) b \), i.e., \( x^* \) is between optimistic and pessimistic scenarios of required emissions reductions with weights defined by ratio of costs \( C_1 \) and \( C_2 \).

3.2 Comparative analysis

The Proposition of Section 3.1 allows the comparison of cases of perfect information, full uncertainty, and uncertainty with learning. The optimal condition given by equation (6) defines a quantile of the underlying probability distribution, i.e., it shows the critical dependence of period 1 ex-ante optimal decision on the probability distribution \( H \). Assume that \( C_1 < C_2 \). In the
case of perfect information, i.e., when $\theta$ is known at the beginning of the first period, both $x_1$ and $x_2$ can be chosen as a function of observable $\theta$. Clearly, the optimal solution is $x_1^* = \theta$, $x_2^* = 0$, i.e., the term $C_1 \theta$ of the first equation in (7) represents indeed the expected cost under perfect information (assuming $\theta = \theta$, i.e., uncertainty is unbiased with respect to the true state of the world). The second term represents the expected value of perfect information because this cost would be eliminated if $\theta$ were known before the first-period emission reduction decision had to be made (rather than afterward as in the learning case). In the case of full uncertainty ("without learning"), the optimal decision is $x_1^* = \overline{\theta}$, $x_2^* = 0$, which is also known as the certainty equivalent. The possibility of learning combined with explicit introduction of ex-post decisions specifies optimal period 1 abatements by the quantile satisfying (6). It may exceed the certainty equivalent $\theta$ or it may be below this level. As equation (6) shows, this depends on the relative values of costs $C_1, C_2$, and the probability distribution $H$. For example, if $C_1/C_2 = 1/2$ and $\theta$ has a normal distribution, then optimal ex-ante abatement coincides with the certainty equivalent $x_1^* = \theta$. For non-symmetric probability distributions, the optimal abatements can be below or above $\theta$. An asymmetric probability distribution can be caused, for example, by the interaction of a symmetric probability distribution with an environmental constraint. For example, if the probability density function for $e(\omega)$ is normal, the distribution for $\theta(\omega)$ can still be asymmetric if there is an atmospheric concentration constraint that does not require emissions reductions for all $\omega$.

**Remark 4.** The certainty equivalent solution $x_1^* = \theta$, $x_2^* = 0$ in the case of full uncertainty (no learning) does not satisfy (4) for all $\theta$, which may lead to a catastrophic collapse of high probability. The only way to fulfill the safety constraint (4) is to choose $x_1$ from the worst case scenario as $x_1 = \max_\omega \theta(\omega), \omega \in \Omega$. Clearly, this is an unrealistic and extremely costly solution. This calls for the explicit introduction of safety constraint (2) to provide a trade-off between the cost effectiveness and risk. The optimal solution under full uncertainty is now defined as minimizing $C_1 x_1 + C_2 x_2$ under constraint $x_1 + x_2 \geq z_1$, where $z_1$ is the minimal $z$ satisfying equation $P[z \geq \theta] = 1 - \gamma$, i.e., as $x_1 = z_1$ and since $C_1 < C_2$, $x_2 = 0$. Clearly, the risk-adjusted solution under full uncertainty $x_1 = x_1$ may be greater or less than $\overline{\theta}$, depending on $\gamma$, $C_1/C_2$, and probability distribution $H$.

### 3.3 Discussion

To summarize the key results, we find that even in this basic model, the effect of learning on optimal first stage emissions reductions is ambiguous; it can lead to either more or less emissions reductions than would be undertaken if there were uncertainty but no learning, and the direction and magnitude of this effect depends on the relationship between the assumed marginal costs in stages 1 and 2, and the shape of the probability distribution characterizing the uncertainty in total required emissions reductions.
The nature of the optimal solution with learning is not counter-intuitive. Whether to hedge against the possibility that required reductions will be large (or small) in stage 2 by making larger (or smaller) reductions in stage 1 depends on expectations about the tradeoffs in reduction costs (how much cheaper, or more expensive, will it be to make reductions if they are postponed) and about the likelihood that exceptionally large or small reductions will turn out to be required. For example, if marginal reduction costs are known to be cheaper in stage 2, it is always best to postpone all reductions however large they might be to stage 2. If, however, marginal costs are cheaper in stage 1, then the advantages of these cheaper costs must be balanced against the risk that you will reduce emissions in stage 1 more than turns out to be necessary. This tradeoff takes on a simple functional form in this model: it is optimal in stage 1 to reduce emissions up to the quantile of the uncertainty distribution for total emissions reductions given by the ratio of marginal costs between the two periods. If costs are twice as high in stage 2, it is optimal to reduce in stage 1 up to the median of the uncertainty distribution; if costs are three times as high in stage 2, it is optimal to reduce up to the 33-rd percentile, etc.

Clearly, then, a wide range of solutions are possible depending on the particular ratio of costs and the shape of the uncertainty distribution. In comparison, the solution under uncertainty without learning also depends on the costs and the shape of the uncertainty distribution, but there is no interaction between the two. All reductions are made either in stage 1 or in stage 2, depending on when marginal costs are lower. The amount of reductions made depends on the certainty with which the constraint is desired to be met. If a decision maker wants to be 50 percent sure the constraint is achieved, reductions to the median of the distribution will be made. Thus it is easy to see that no particular relationship between the solution with learning and without need hold. Depending on assumptions, optimal reductions with learning can be smaller, larger, or the same as in the no learning case, even in this simple linear model.

4 Extensions to the Basic Model

Next we examine separately three aspects of the basic concentration-stabilization model with learning presented in Section 3 that can be considered oversimplified. First, we replace the linear cost assumption with nonlinear abatement costs. We show that assuming costs are quadratic in reductions implies that it will always be optimal to make at least some reductions during stage one, a stronger result than occurs in the linear case. We also show that, except in the case of normally distributed uncertainty, the optimal solution will not depend only on the mean value of the uncertainty distribution. Thus deterministic analyses that use only the mean value will be misleading. Second, we assume that there is a limited capacity for making emissions reductions in
stage two, an assumption that could be motivated by inertia in technological or socio-economic systems, or by the possibility that stage one (the period before learning occurs) may be long, leaving little time to make reductions in stage two. We show that the assumption of limited adaptive capacity in stage two induces greater optimal emissions reductions in stage one. Third, we assess the implications of incomplete learning, i.e., learning in which uncertainty is not completely resolved before stage two begins. We show that, as in the case of complete learning, the effect of learning on optimal stage one decisions is ambiguous: it can lead to larger, smaller, or the same emissions reductions as would be made if there were no learning. The effect depends on the assumed marginal costs in the two periods, and the nature and likelihood of what might be learned.

4.1 Nonlinear abatement costs

Abatement costs are generally modeled as nonlinear functions of emission reductions [12] with a quadratic functional form of a typical assumption (e.g., [20]). To examine the implications of this assumption, we let the cost functions of both periods be of the form \( C_i(x) = C_i x^2 \) with positive \( C_1, C_2 \). Cost function (5) then takes on the form \( F(x) = C_1 x^2 + C_2 E(\max\{0, \theta - x\})^2 \) and hence \( F'(x) = 2C_1 x - 2C_2 E(\theta - x) I[\theta \geq x] \). The first observation we make concerns the necessity of first period reductions. Since \( F'(0) = -2C_2 E\theta < 0 \) (assuming that \( E\theta > 0 \)) zero reductions in stage one are ruled out independently of the particular values of \( C_1 \) and \( C_2 \). Compare this results to the case of linear costs in Section 3.1, where \( F'(0) = C_1 - C_2 < 0 \) if \( C_1 < C_2 \); i.e., non-zero first period reductions are called for only if costs are less in period 1. With quadratic costs, period 1 reductions are optimal even if \( C_1 > C_2 \).

Let us illustrate some typical situations that may occur in the case of non-smooth, piece-wise linear functions commonly used in emission-control problems with technology switches. These functions implicitly impose upper or lower bounds on ex-ante emission reductions. Assume that \( C_2(x) = C_2 x \) and \( C_1(x) \) is a piece-wise linear function \( C_1(x) = C_1^0 x \) for \( 0 \leq x \leq a \) and \( C_1(x) = C_1^2 (x-a) + C_1^3 a \) for \( x \geq a \), where \( C_1^0 < C_2 \) and \( C_1^3 > C_2 \). It is easy to see that the optimal ex-ante solution has the upper bound \( x_1 \leq a \).

As \( C_1^0 < C_2 \) and \( C_1^3 > C_2 \), the optimal ex-ante decision is defined as follows: let \( \bar{\pi} \) be the solution of equation \( P[\theta \geq x] = C_1/C_2 \). The optimal period 1 decision \( x_1^* = a \) if \( \bar{\pi} > a \), and \( x_1^* = \bar{\pi} \) for \( \pi \leq a \). Assume that \( C_1(x) = C_1 x \), and \( C_2(x) = C_2^0 x \) for \( 0 < x \leq a \); \( C_2(x) = C_2^2 (x-a) + C_2^3 a \) for \( x \geq a \), where \( C_1 > C_2^0 \), \( C_1 < C_2^2 \). Consider solution \( \pi \) of the equation \( P[\theta > x] = C_1/C_2^3 \). It is easy to see that the optimal period 1 decision \( x_1^* = \pi \) for \( \pi \geq a \) and \( x_1^* = 0 \) for \( \pi < a \), i.e., it has the lower bound \( x_1 \geq a \).
4.2 Limited adaptive capacity

We next introduce the possibility that the capacity for emissions abatement in stage two may be limited. There are several possible motivations for this assumption. First, in cases with learning, Learning may occur slowly and the second period may begin late, leaving little time for reductions to be made. Second, inertia in technological and socio-economic systems may limit feasible reductions over a given time period \[12\]. The path-dependencies (inertia) of the technological and socio-economic systems producing greenhouse gases are critical for dealing with abrupt changes. Without inertia, the switching from one emission path to another would be instantaneous. In reality, energy production systems cannot be changed overnight. As a result, the possibilities for emissions reductions will not be bounded.

Limited adaptive capacity can be modeled most simply by constraints \[x_2 \leq \beta\] with positive random \(\beta\) which becomes known from learning at stage 2. Without the safety constraint of type (2), the optimal stage 2 decision \[x_2 = \min \{\beta, \max \{0, \theta - x_1\}\}\] cannot in general satisfy safety constraints (4) for all \(\theta\). As a consequence, the probability of a catastrophe can be rather high, calling for explicit introduction of type (2) safety constraint \[P[x_1 + x_2 \geq \theta] = 1 - \gamma\]. Since \(x_2 \leq \beta\), this requires ex-ante emission reduction commitments \(x_1 \geq x_\gamma\), where \(x_\gamma\) is minimal non-negative \(x\) satisfying equation \[P[x \geq \theta - \beta] = 1 - \gamma\]. Therefore, in order to prevent a catastrophic collapse with sufficient confidence, there must be minimal ex-ante emission reductions sufficient to satisfy the safety constraint in stage 2. Hence, stage one emission abatement is in general larger when limited adaptive capacity is explicitly assumed than when possible future emissions reductions are assumed to be unbounded. This can be evaluated properly by analyzing the STO model with safety constraints (2).

4.3 Incomplete learning and safety constraint

Next we replace the assumption that uncertainty is completely resolved before stage two with the much more realistic assumption that learning is only partial. As an example, we consider the case in which learning affects the prior distribution, \(H(z)\), by shifting the range of uncertainty. We first present the quantitative analysis, then discuss the results in qualitative terms.

Let us assume that \(H(z) = P[\theta \leq z]\) is a mixture \(H(z) = E_{z} H(\xi, z) = \int H(y, z) dG(y)\) of distribution \(H(\xi, z)\) with unknown \(\xi\) characterized by a probability distribution \(G(y) = P[\xi \leq y]\). The learning reveals only \(\xi\) at the beginning of period 2. For example, \(H(z)\) can be a mixture of distributions \(H(\xi, z)\) with probability mass concentrated in different subregions from the support of \(H(z)\); in reality, these distributions could reflect differing views on the damages that would be associated with particular emissions pathways. (Note that if the support of \(H(\xi, z)\) is a singleton, then the learning of \(\xi\) reveals the true value of \(\theta\). For the sake of illustration, let \(H(z)\) be a mixture
of two distributions \( H_0(z) \) and \( H_1(z) \), that is, \( \xi H_0(z) + (1 - \xi) H_1(z) \), where \( \xi = 0 \) with probability \( p \) and \( \xi = 1 \) with probability \( 1 - p \), that is, \( H(z) = pH_0(z) + (1 - p)H_1(z) \). Since only \( \xi \) is observed, the period 2 decision \( x_2(\xi) \) cannot fulfill constraints (4), and the safety constraint has to be written as in (2):

\[
P[x_1 + x_2(\xi) \geq \theta(\xi)] \geq 1 - \gamma, \xi = 0, 1,
\]

where \( \theta(\xi) \) has the posterior probability distribution \( H_\xi(z) \). For a given \( \xi \) and \( \gamma \), let us define \( z_\gamma(\xi) \) as the minimal \( z \), satisfying equation \( P[z \geq \theta(\xi)] = 1 - \gamma \). Then equations (8) are equivalent to the equations \( x_1 + x_2(\xi) \geq z_\gamma(\xi) \), which are similar to (4). The optimal period 2 decision \( x_2(\xi) = \max \{0, z_\gamma(\xi) - x_1\} \).

Function \( F(x) \) does not have continuous derivatives. Therefore, the optimality condition cannot be derived from the Proposition of Section 3.1. \( F(x) \) is a piece-wise continuous linear function which can be characterized as the following. Assume, for example, that \( z_\gamma(0) < z_\gamma(1) \), then for \( 0 \leq x < z_\gamma(0) \),

\[
F(x) = C_1 x + C_2 [p(z_\gamma(0) - x) + (1 - p)(z_\gamma(1) - x)] = (C_1 - C_2) x + C_2 z_\gamma(\xi).
\]

For \( z_\gamma(0) \leq x < z_\gamma(1) \), \( F(x) = C_1 x + C_2 (1 - p)(z_\gamma(1) - x) = (C_1 - C_2 (1 - p))x + C_2 (1 - p)z_\gamma(1) \), and for \( x \geq z_\gamma(1) \), \( F(x) = C_1 x \).

The optimal ex-ante solution hedges against the different contingencies. It is characterized as follows: \( x = 0 \), if \( C_1 > C_2 \). Otherwise, \( x = z_\gamma(0) \), if \( C_1 - C_2 (1 - p) > 0 \), and \( x = z_\gamma(1) \), if \( C_1 - C_2 (1 - p) < 0 \).

This solution can be understood in more qualitative terms as follows. If marginal costs are lower in stage two, then it is best to make all reductions in stage two after learning has taken place. If marginal costs are lower in stage one, then, in general it pays to make reductions in period 1 that are as large as possible. However, as was the case in the basic model in Section 3, this benefit of period 1 reductions must be weighed against the risk of making more reductions than turn out to be required. After learning takes place at the end of period 1, the optimal solution is to make reductions such that the total reduction is either \( z_\gamma(0) \) or \( z_\gamma(1) \). Thus the minimal first period reduction is \( z_\gamma(0) \). If first period costs are very low, or the chance that \( \xi = 1 \) is very high, then it is optimal to make the larger first period reduction \( z_\gamma(1) \), accepting the chance that \( \xi = 0 \) and that reductions \( z_\gamma(1) - z_\gamma(0) \) will have been unnecessary.

Let us compare this ex-ante period 1 optimal "with-learning" solution to the optimal "without-learning" solution \( x_1^* = z_\gamma \), \( x_2^* = 0 \) derived from minimization of (3) under safety constraint \( P[x_1 + x_2 \geq \theta] \geq 1 - \gamma \), i.e., \( x_1 + x_2 \geq z_\gamma \), where \( z_\gamma \) is the minimal \( z \) satisfying constraint \( P[z \geq \theta] = 1 - \gamma \). Due to the monotonicity of \( P[x \geq \theta] \) w.r.t. \( x \), we can derive inequalities among these decisions by comparing \( P[x \geq \theta] \) for \( x = z_\gamma \) with \( x = z_\gamma(0), z_\gamma(1) \).

Assume that \( H_0(z) \), \( H_1(z) \) have continuous derivatives, the support of distribution \( H_\theta(z) \) is interval \([a_\delta, b_\delta]\), and the support of \( H_1(z) \) is interval \([a_1, b_1]\), where \( a_1 > b_\delta \). If \( C_1 - C_2 (1 - p) < 0 \), then the optimal "with-
learning" period 1 solution \( x = z_1(1) \) from \([a_1, b_1]\). If \( x \in [a_1, b_1] \), then \( P[x \geq \theta] = p + (1 - p) H_1(x) \). Since \( H_1(z_1(1)) = 1 - \gamma \), then \( P[x \geq \theta] = p + (1 - p)(1 - \gamma) = 1 - \gamma + \gamma p \) for \( x = z_1(1) \). As \( \gamma p > 0 \), then the optimal "without-learning" decision \( x = z_1 \) satisfying \( P[x \geq \theta] = 1 - \gamma \) is less demanding (smaller) than \( x = z_1(1) \), i.e., learning increases the optimal ex-ante emission reductions. This conclusion is reversed in the case \( C_1 - C_2(1 + p) > 0 \). Indeed, let \( x \in [a_0, b_0] \). Then \( P[x \geq \theta] = p H_0(x), H_0(z_1(0)) = 1 - \gamma \) and for \( x = z_1(0) \), \( P[x \geq \theta] = (1 - \gamma) \) (i.e., the optimal "without-learning" decision \( x = z_1 \) is greater than the optimal "with-learning" decision \( x = z_1(0) \)).

Therefore we find that in the case of incomplete learning, as in the case of complete learning we assumed in Section 3, the effect of learning is ambiguous: it can lead to either larger, smaller, or the same optimal emissions reductions in stage one as would occur under uncertainty without learning. The particular nature of the effect is determined by the marginal costs of reductions in the two periods, and the assumed likelihood of what will be learned at the start of stage two in the example presented here, the likelihood that one of two competing uncertainty distributions will turn out to be supported by the new information received. The size of the effect is determined by the shape of the distributions themselves, and the certainty with which it is desired to achieve the safety constraint.

5 A Dynamic Stabilization Problem

In this section we extend the two-period model presented in Section 3 to multiple periods. In this more general form, the problem becomes similar to catastrophic-risk-management problems discussed in [8]. As discussed in Section 3, the solution of the two-stage model had strong connections with CVaR-type risk measures. Here we show that the dynamic multi-period model also has strong connections with dynamic versions of CVaR risk measures. However, we caution that this resemblance may create the impression of a truly risk-based policy analysis when in fact, without the explicit introduction of additional safety constraints, the solution could provoke a catastrophic collapse.

Assume that CO\(_2\) emission paths are characterized by exogenous scenarios as in Section 3. Let us consider \( R_t = \sum_{k=0}^t x_k \), where decision variables \( x_k \geq 0, k = 0, 1, ..., t, t \leq T \). We can think of \( x_k \) as a feasible level of CO\(_2\) emission reduction at the beginning of period \( k \). At time \( t = 0, 1, ..., \), the target value on total emission reduction \( R_t \) in period \( t \) is given as a random variable \( \rho_t \). It is assumed that exact value of \( \rho_t \) is revealed at a random time \( t = \tau \). Since \( \tau \) is uncertain, the decision path \( x = (x_0, x_1, ..., x_T) \) for the whole time horizon has to be chosen ex-ante in period \( t = 0 \) to "hit" the target \( \rho_t, R_\tau \geq \rho_\tau \), at \( t = \tau \) in a sense specified further by (10). At random \( t = \tau \), the decision path can be revised for the remaining available time. Similar to the model of Section 3.1, consider a stream of linear random
costs \( v(x) = \sum_{t=0}^{T} c_t x_t + \delta \max \{0, \rho_t - R_t\} I(\tau = t) \), where \( c_t > 0, \delta > 0 \), \( t = 0, 1, \ldots, T \) are known ex-ante and ex-post abatement costs. The expected value of costs \( v(x) \) can be written as

\[
V(x) = \sum_{t=0}^{T} c_t x_t + Ed_t \max \left\{ 0, \rho_t - \sum_{t=0}^{\tau} x_t \right\}.
\] (9)

Let us consider a path \( x^* \) minimizing \( V(x) \) subject to \( x_t \geq 0, t = 0, 1, \ldots, T \). Assume that \( V(x) \) is a continuously differentiable function (e.g., a component of random vector \( \rho = (\rho_0, \rho_1, \ldots, \rho_T) \) has a continuous density function). Assume also that, so far, there exist positive optimal solution \( x^* = (x_0^*, x_1^*, \ldots, x_T^*) \), \( x_t^* > 0, t = 0, 1, \ldots, T \). Then from the optimality condition for stochastic maximax problems similar to Section 3, it follows that for \( x = x^* \),

\[
V_{x_t} = c_t - \sum_{k=0}^{t} p_k d_k P[\sum_{k=0}^{t} x_k \leq \rho_t] = 0, t = 0, 1, \ldots, T,
\]

where \( p_k \) is the probability that \( \tau \) occurs first time at \( t \). From this sequentially for \( t = 0, 1, \ldots, T \), it follows that

\[
P[\rho_0 \leq \rho_t] = c_0/p_0 d_0, P[\sum_{k=0}^{t} x_k \leq \rho_t] = (c_t - c_{t-1})/p_t d_t, t = 1, \ldots, T. \tag{10}
\]

From (10) it follows that

\[
V(x^*) = c_0 E[\rho_0 I(\rho_0 > R_0^*)] + (c_1 - c_0) E[p_1(\rho_1 > R_1^*)] + \ldots + (c_T - c_{T-1}) E[p_T(\rho_T > R_T^*)],
\]

which can be viewed as a dynamic CVaR risk measure.

Remark 6. Equations (10) are derived from the existence of the positive optimal solution \( x^* \). It is easy to see that the existence of this solution follows from \( c_0/p_0 d_0 < 1, (c_t - c_{t-1})/p_t d_t < 1 \) and the monotonicity of quantiles \( \beta_t \), \( \beta_0 < \beta_1 < \ldots < \beta_T \) defined by equations

\[
P[\beta_0 < \rho_t] = c_0/p_0 d_0, P[\beta_t < \rho_t] = (c_t - c_{t-1})/p_t d_t, t = 0, 1, \ldots, T.
\]

If probability \( p_t \) rapidly decreases to 0, e.g., if \( p_t \) is associated with a rare catastrophic event, then from (10) it follows that ex-ante abatements are positive for a relatively short initial interval defined by inequality \( (c_t - c_{t-1})/p_t d_t < 1 \). This misleading conclusion is due to a strong assumption of unlimited capacity for emission reductions, which is a standard assumption of climatic-economic integrated assessment models (see discussions in [12] and [28]). Similar to conclusions of Section 4.2, this requires an adequate treatment of risks by additional safety constraints (2) to prevent catastrophes.
6 Concluding Remarks

This paper analyzes the effects of risks and learning on climate change decisions using a two-stage, dynamic model that assumes a concentration-stabilization constraint. It shows that learning can lead either to larger or smaller first period emissions reductions, compared to the optimal reduction under uncertainty without learning, and that this effect can either be large or small. The direction and magnitude of the learning effect is determined by a number of interacting factors. For example, in a simple linear model with deterministic mitigation costs but uncertainty in total required emissions reductions, the learning effect depends on how mitigation costs evolve over time, the shape of the uncertainty distribution in required emissions reductions, the confidence with which the safety constraint (i.e., stabilization level) is desired to be met, and, in the case of incomplete learning, the probability distribution describing the anticipated learning possibilities. Introducing a more realistic nonlinear cost function with increasing marginal costs induces a higher level of first period emissions reductions compared to the linear case. We also analyze the case of limited capacity to make reductions in period 2, motivated by either uncertain timing of learning or uncertain inertia in socio-economic systems, and show how this consideration can induce a minimum level of first period reductions. Finally, framing the problem in dynamic terms as a multi-period problem with an uncertain time path of required cumulative emissions reductions shows that the problem has strong connections with dynamic versions of CVaR risk measures. This may create the misleading impression that risks are being properly managed, and unless additional safety constraints are introduced, could provoke a catastrophe. Given the multiple influences on the learning effect, we conclude that drawing practical conclusions on the likely effect of learning on climate change decisions is an empirical question requiring analysis with models capable of adequately representing endogenous risks, abrupt changes, realistic learning rates, inertia, and path dependent costs.

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